1. You have a job as a software engineer at BC Hydro. During the yearly safety review it is revealed that certain system-critical pieces of software have not been verified for correctness at the nuclear power plant along the Skeena river. For example, there is an unverified procedure that plays an important role in managing the operation of the control rods. Every 12 seconds a certain number of control rods are inserted into the uranium bundles to slow the reaction. The number of control rods to be inserted is determined by the current rate of the reaction. However, the choice of which control rods to be inserted is computationally complex and the optimal solution is found by testing all possibilities. Every possibility can be represented by a binary string with a fixed number of 1s (representing control rods that are inserted) and a fixed number of 0s (representing control rods that are not inserted). In other words, each possibility can be represented by an \((s, t)\)-combination. Your job during this year’s safety review is to verify that the following ControlRods program does in fact generate all possible \((s, t)\)-combinations.

```
ControlRods(s, t)
Require: s, t > 0
1: b := array(01^0^s−1)
2: x := 2
3: y := 1
4: test(b)
5: while x < s + t do
 6: b[x] := 0
 7: b[y] := 1
 8: b[1] := b[x + 1]
 9: b[x + 1] := 1
10: x := x + 1 − (x − 1) * b[2] * (1 − b[1])
11: y := b[1] * y + 1
12: test(b)
13: end while
14: b[x] := 0
15: b[y] := 1
16: test(b)
```

An important step in verifying that a program is correct is understanding how it works. In the program every \((s, t)\)-combination gets tested exactly once by the test routine that is called on lines 4, 12, and 16. To understand how the program works you decide to replace the test(b) statements by print(b,y,x). The resulting output for \(s = 3\) and \(t = 3\) appears in the table above. Answer the following questions in plain English.

a) What does the value of \(y\) represent? [5 marks]
b) What does the value of \(x\) represent? [5 marks]
c) What order are the values of \(b\) generated in? [5 marks]

d) Rewrite line 11 using an appropriate if statement. [5 marks]
e) Rewrite line 10 using an appropriate if statement. [5 marks]
2. Consider the following two programs.

**Copy** \((x, y)\)

1: \(a := x\)
2: \(y := 0\)
3: \(\text{while } a <> 0 \text{ do}\)
4: \(y := y + 1\)
5: \(a := a - 1\)
6: \(\text{end while}\)

**Multiply** \((x, y)\)

1: \(a := 0\)
2: \(z := 0\)
3: \(\text{while } a <> y \text{ do}\)
4: \(z := z + x\)
5: \(a := a + 1\)
6: \(\text{end while}\)

a) Give a loop invariant that would allow you to prove \(|\{ x \geq 0 \}\)Copy\((x, y)\{x = y\}|. [5 marks]

b) Give a loop invariant that would allow you to prove \(|\{ y \geq 0 \}\)Multiply\((x, y)\{z = x \cdot y\}|. [5 marks]

3. Consider the following program.

1: \(y := x\)
2: \(\text{while } y > 0 \text{ do}\)
3: \(y := y - 2\)
4: \(\text{end while}\)
5: \(\text{if } y = 0 \text{ then}\)
6: \(z := 0\)
7: \(\text{else}\)
8: \(z := 1\)
9: \(\text{end if}\)

a) Verify partial correctness for precondition \(\{ x > 0 \}\) and postcondition \(\{ z = x \mod 2 \}\). [10 marks]

b) What is the weakest precondition that would ensure total correctness? [5 marks]

4. Using the techniques from the textbook (Chapter 9) answer the following questions.

a) What is the value of \(wp(x:=x+y; y:=x*y, x < y)? [5 marks]\n
b) What is the value of \(wp(\text{if } x=0 \text{ then } y:=y-1 \text{ else } y:=0 \text{ end, } x = y)? [5 marks]\n