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Abstract
Combinatorial objects are represented by strings, such as 21534 for the permutation (1 2) (3 4 5), or 11010 for the binary tree corresponding to the balanced parentheses ( ( ) ( ) ). Given a string $s=s_1s_2...s_n$, the right-shift operation $\text{rshift}(s, i, j)$ replaces the substring $s_is_{i+1}...s_j$ by $s_{i+1}...s_js_i$, where $1 \leq i < j \leq n$. In other words, $s_i$ is right-shifted into position $j$ by applying the permutation $(j \ j–1 \ ... \ i)$ to the indices of $s$. Right-shifts include prefix-shifts ($i=1$) and adjacent-transpositions ($j=i+1$). A fixed-content language is a set of strings that contain the same multiset of symbols. Given a fixed-content language, a shift Gray code is a list of its strings where consecutive strings differ by a shift. For example, in a right-shift Gray code, each $s$ is followed by some $\text{rshift}(s, i, j)$.

This thesis uncovers the first prefix-shift Gray code for multiset permutations, as well as the first $O(1)$-time algorithm using $O(1)$ additional variables for generating them. Applications of these basic results include more efficient exhaustive solutions to stacker-crane problems, which are NP-complete traveling salesman variants requiring movement along specified arcs. This thesis also uncovers a new fastest algorithm for generating balanced parentheses, and the first minimal change order for fixed-content necklaces and Lyndon words.

These results are consequences of the following theorem: Every bubble language has a right-shift Gray code. Bubble languages are fixed-content languages that are closed under certain adjacent-transpositions. These languages generalize classic combinatorial objects — k-ary trees, ordered trees with fixed branching sequences, unit interval graphs, restricted Schröder and Motzkin paths, linear-extensions of B-posets — and their unions, intersections, and quotients. Each Gray code is circular and is obtained by creating a new variation of lexicographic order known as cool-lex order.

Shorthand universal cycles are universal cycles for fixed-content languages that omit the last (redundant) symbol from each substring. When the missing symbol is restored, the strings appear in a circular Gray code using only $\text{rshift}(s, 1, n)$ and $\text{rshift}(s, 1, n–1)$. This thesis provides the first construction for multiset permutations. When applied to binary strings, the result is a new fixed-density analogue to classic de Bruijn cycles, and is also the first universal cycle for the “middle levels” (binary strings of length 2k+1 with sum k or k+1).

Awards, Scholarships, Fellowships
2008 Best Student Paper (CATS 08)
2004-2007 NSERC Postgraduate Scholarship Doctoral
2003 Teaching Assistant Award (Waterloo C&O)
2002-2003 OGS Ontario Graduate Scholarship

Publications

Ruskey, F., and Williams, A. An Explicit Universal Cycle for the (n-1)-Permutations of an n-Set. ACM Transactions on Algorithms. Accepted.

Ruskey, F., and Williams, A. The Coolest way to Generate Combinations. Discrete Mathematics.