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University of Victoria

Programme

Faculty of GRADUATE STUDIES
Thinking Outside the box

The Final Oral Examination
for the Degree of

DOCTOR OF PHILOSOPHY
Department of Computer Science

Aaron Williams

2001 B.Math University of Waterloo
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"Shift Gray Codes"

October 23rd, 2009
9:00 am
Engineering/Computer Science Bldg. (ECS), Room 660

Supervisory Committee:
Dr. Frank Ruskey, Department of Computer Science, UVic (Co-Supervisor)
Dr. Wendy Myrvold, Department of Computer Science, UVic (Co-Supervisor)
Dr. Ulrike Stege, Department of Computer Science, UVic
Dr. Peter Dukes, Department of Mathematics and Statistics, UVic (Outside Member)

External Examiner:
Dr. Ronald Graham, University of California

Chair of Oral Examination:
Dr. Laurel Bowman, Department of Greek & Roman Studies, UVic

Abstract

Combinatorial objects are represented by strings, such as 21534 for the permutation (1 2) (3 4 5), or 110100 for the binary tree corresponding to the balanced parentheses (( ) ( )). Given a string $\mathbf{s}=s_1s_2...s_n$, the right-shift operation $\mathrm{rshift}(\mathbf{s},i,j)$ replaces the substring $s_is_{i+1}...s_j$ by $s_{i+1}...s_js_i$, where $1\le i<j\le n$. In other words, s_i is right-shifted into position j by applying the permutation (j j−1 ... i) to the indices of **s**. Right-shifts include prefix-shifts (i=1) and adjacent-transpositions (j=i+1). A fixed-content language is a set of strings that contain the same multiset of symbols. Given a fixed-content language, a shift Gray code is a list of its strings where consecutive strings differ by a shift. For example, in a right-shift Gray code, each **s** is followed by some $\mathrm{rshift}(\mathbf{s},i,j)$.

This thesis uncovers the first prefix-shift Gray code for multiset permutations, as well as the first O(1)-time algorithm using O(1) additional variables for generating them. Applications of these basic results include more efficient exhaustive solutions to stacker-crane problems, which are NP-complete traveling salesman variants requiring movement along specified arcs. This thesis also uncovers a new fastest algorithm for generating balanced parentheses, and the first minimal change order for fixed-content necklaces and Lyndon words.

These results are consequences of the following theorem: Every bubble language has a right-shift Gray code. Bubble languages are fixed-content languages that are closed under certain adjacent-transpositions. These languages generalize classic combinatorial objects — k-ary trees, ordered trees with fixed branching sequences, unit interval graphs, restricted Schröder and Motzkin paths, linear-extensions of B-posets — and their unions, intersections, and quotients. Each Gray code is circular and is obtained by creating a new variation of lexicographic order known as cool-lex order.

Shorthand universal cycles are universal cycles for fixed-content languages that omit the last (redundant) symbol from each substring. When the missing symbol is restored, the strings appear in a circular Gray code using only $\mathrm{rshift}(\mathbf{s},1,n)$ and $\mathrm{rshift}(\mathbf{s},1,n-1)$. This thesis provides the first construction for multiset permutations. When applied to binary strings, the result is a new fixed-density analogue to classic de Bruijn cycles, and is also the first universal cycle for the "middle levels" (binary strings of length 2k+1 with sum k or k+1).

Awards, Scholarships, Fellowships

2008	Best Student Paper (CATS 08)
2004-2007	NSERC Postgraduate Scholarship Doctoral
2003	Teaching Assistant Award (Waterloo C&O)
2002-2003	OGS Ontario Graduate Scholarship

Publications

Williams, A. *Loopless Generation of Multiset Permutations by Prefix Shifts.* SODA (Symposium on Discrete Algorithms), New York, United States, 2009.

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